Department of Mathematics & Philosophy of Engineering

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Course: MPZ 3231-Engineering Mathematics IA **Model Answer No·04**

Academic Year – 2013/2014

1. a) let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$

$$\frac{z_1}{z_2} = \frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} = \frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)}\,\frac{(\cos\theta_2 - i\sin\theta_2)}{(\cos\theta_2 - i\sin\theta_2)}$$

$$\therefore \frac{z_1}{z_2} = \frac{r_1}{r_2}\,\frac{(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) + i(\sin\theta_1\cos\theta_2 - \cos\theta_1\sin\theta_2)}{(\cos^2\theta_2 + \sin^2\theta_2)}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\{\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\}$$

$$\therefore \arg\frac{z_1}{z_2} = (\theta_1 - \theta_2) \qquad \theta_1 = \arg z_1\,, \theta_2 = \arg z_2$$

$$\arg\frac{z_1}{z_2} = \arg z_1 - \arg z_2$$

$$z_1 = -1 + i = \sqrt{2}\left[\frac{-1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right] = \sqrt{2}\left[\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right]$$

$$z_2 = 1 + \sqrt{3}\,i = 2\left[\frac{1}{2} + i\frac{\sqrt{3}}{2}\right] = \sqrt{2}\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right]$$

$$\frac{z_1}{z_2} = \frac{\sqrt{2}}{2}\left\{\cos\left(\frac{3\pi}{4} - \frac{\pi}{3}\right) + i\sin\left(\frac{3\pi}{4} - \frac{\pi}{3}\right)\right\}$$

$$= \frac{1}{\sqrt{2}}\left\{\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right\}$$

$$\frac{z_1}{z_2} = \frac{-1 + i}{1 + \sqrt{3}\,i} = \frac{(-1 + i)(1 + \sqrt{3}\,i)}{4} = \left(\frac{\sqrt{3} - 1}{4}\right) + i\left(\frac{1 + \sqrt{3}}{4}\right)$$

$$\operatorname{Re}\frac{z_1}{z_2} = \frac{1}{\sqrt{2}}\cos\frac{5\pi}{12} = \frac{\sqrt{3} - 1}{4} \qquad \therefore \cos\frac{5\pi}{12} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\operatorname{Im}\frac{z_1}{z_2} = \frac{1}{\sqrt{2}}\sin\frac{5\pi}{12} = \frac{1 + \sqrt{3}}{4}$$

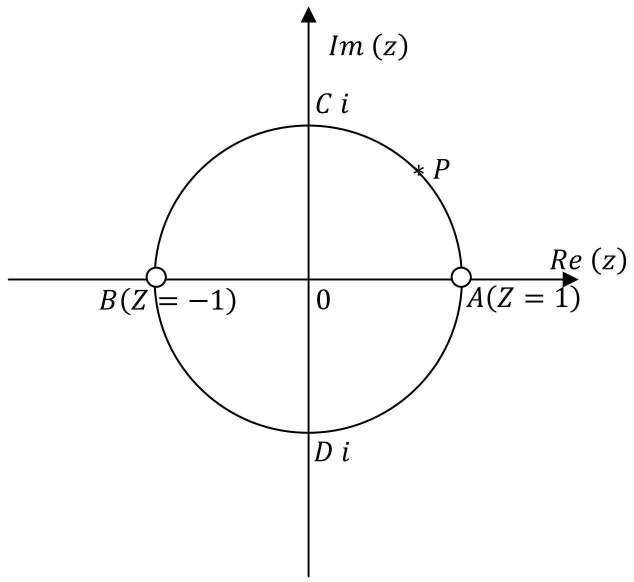
b) $z = x + iy$ where $x, y \in \mathbb{R}$

$$\frac{z-1}{z+1} = \frac{(x-1)+iy}{(x+1)+iy} = \frac{[(x-1)+iy][(x+1)-iy]}{[(x+1)+iy][(x+1)-iy]}$$

$$= \frac{(x^2-1)+y^2}{(x+1)^2+y^2} + i\,\frac{y(x+1-x+1)+y^2}{(x+1)^2+y^2}$$

$$= \frac{x^2+y^2-1}{(x+1)^2+y^2} + i\,\frac{2y}{(x+1)^2+y^2}$$

$$\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2} \qquad \therefore Re\left(\frac{z-1}{z+1}\right) = 0$$



The arch ACB (without the point A and B) representing the locus of P, such that

$$\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$$

The arch ADB (without the point A and B) representing the locus of P, such that

$$\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2}$$

$$(z-1)(z+1) = [(x-1)+iy][(x+1)+iy]$$

$$= (x^2-1-y^2) + iy(x+1+x-1)$$

$$= (x^2-y^2-1) + i2xy$$

$$\arg(z-1)(z+1) = \pi \qquad \therefore Im\ (z-1)(z+1) = 0$$

$$xy = 0$$

If $y = 0$ z is real $\therefore z = 0$ and $|Z| < 1$

Then the line AB (without point A B)

Representing the locus p such that $\arg(z-1)(z+1) = \pi$

If $x = 0$ z is purely imaginary $\therefore z = iy$

$$(z-1) = -1 + iy$$

$$(z+1) = 1 + iy$$

$$(z-1)(z-1) = (-1+iy)(1+iy)$$

$$= -(y^2+1)$$

$\arg(z-1)(z+1) = \pi$ for all values of y

$\therefore$The locus of P is the y_{ODCD}

2. Let $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$ where $x_1, y_1, x_2, y_2 \in \mathbb{R}$

$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$

$|z_1 - z_2|^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$

$$= ({x_1}^2 + {x_2}^2) + ({y_1}^2 + {y_2}^2) - 2(x_1x_2 - y_1y_2)$$

$z_1z_2 = (x_1 + iy_1)(x_2 - iy_2)$

$$= (x_1x_2 + y_1y_2) + i(y_1x_2 - x_1y_2)$$

$x_1x_2 + y_1y_2 = Re\,(z_1\overline{z_2})$ or $Re\,(\overline{z_1}z_2)$

$|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2Re\,(z_1\overline{z_2})$

Since $|z_1| = |z_2| = |z_3|$

$$z_1 = \cos\theta + i\sin\theta$$

$$z_2 = \cos(\theta+\alpha) + i\sin(\theta+\alpha)$$

$$z_3 = \cos(\theta-\beta) + i\sin(\theta-\beta)$$

$z_1 + z_1 + z_1 = 0$

$\cos\theta + \cos(\theta+\alpha) + \cos(\theta-\beta) = 0 - - - (1)$

$\sin\theta + \sin(\theta+\alpha) + \sin(\theta-\beta) = 0 - - - (2)$

From (1)

$$\cos\theta + 2\cos\left(\theta + \frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right) = 0$$

$$\cos\theta + 2\sin\left(\theta + \frac{\alpha-\beta}{2}\right) + \cos\left(\frac{\alpha+\beta}{2}\right) = 0$$

$$\therefore\ \tan\left(\theta + \frac{\alpha-\beta}{2}\right) = \tan\theta$$

$\therefore\ \alpha - \beta = 0 \qquad\qquad \alpha = \beta$

From (2)

$$\cos\theta + 2\cos\theta\cos\beta = 0$$

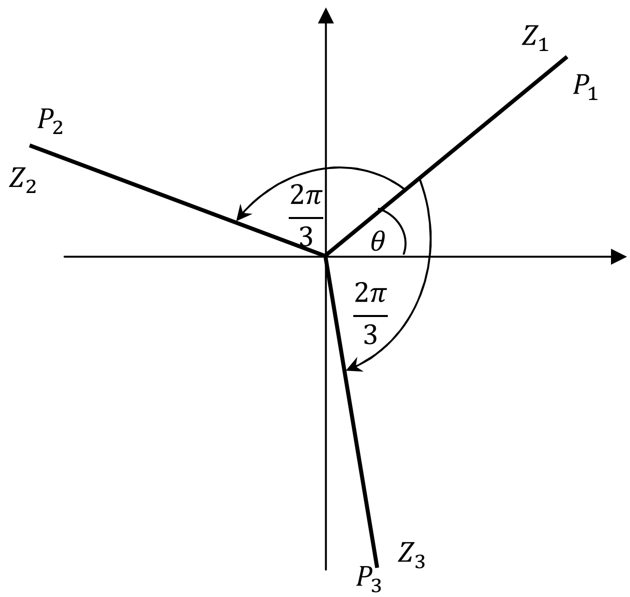
$$\cos\theta \neq 0 \qquad \therefore \cos\beta = -\frac{1}{2} \qquad\qquad \therefore\ \beta = \frac{2\pi}{3}$$

$z_1 = \cos\theta + \ i\sin\theta$

$z_2 = \cos\left(\theta + \frac{2\pi}{3}\right) + \ i\sin\left(\theta + \frac{2\pi}{3}\right)$

$z_3 = \cos\left(\theta - \frac{2\pi}{3}\right) + \ i\sin\left(\theta - \frac{2\pi}{3}\right)$

$P_2OP_3 = \frac{2\pi}{3}$



$P_1P_2P_3$ is an equilateral triangle

b) Let $z = x + iy$ where $x, y \in \mathbb{R}$

$z - 2i = x + (y-2)i$ and $z + 4 = (x+4) + iy$

$$\frac{z-2i}{z+4} = \frac{x+(y-2)i}{(x+4)+iy} = \frac{[x+(y-2)i][(x+4)-iy]}{[(x+4)+iy][(x+4)-iy]}$$

$$= \frac{x(x+4)+y(y-2)}{(x+4)^2+y^2} + i\frac{(y-2)(x+2)-xy}{(x+4)^2+y^2}$$

$$Re\left(\frac{z-2i}{z+4}\right) = 0 \iff x(x+4)+y(y-2) = 0$$

$$x^2+y^2+4x-2y = 0$$

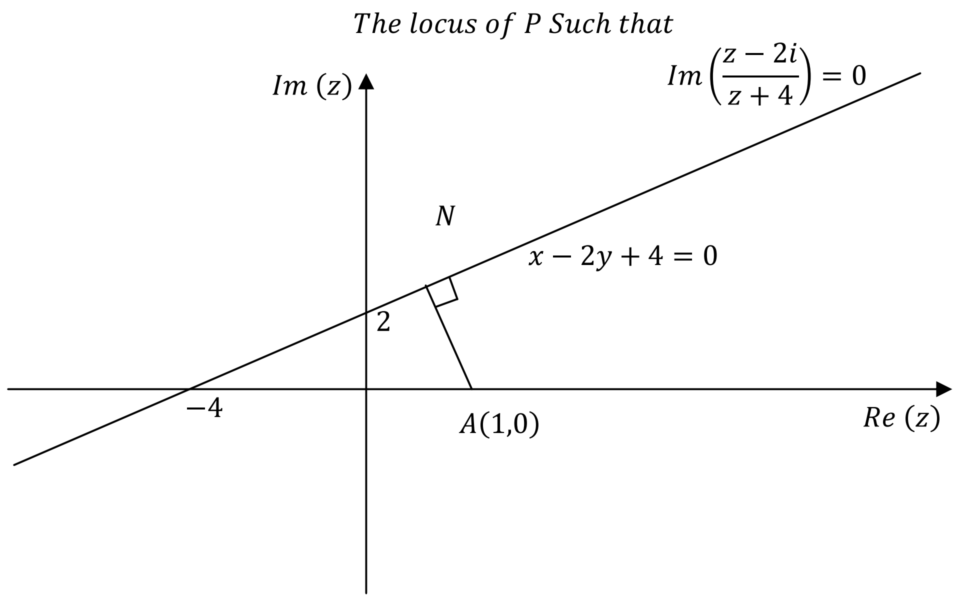
$$(x+4)^2+(y-1)^2-\left(\sqrt{5}\right)^2 = 0$$

$\therefore$ The locus of P is a circle which has the center $(-2,1)$ and the radius $\sqrt{5}$.

$$Im\left(\frac{z-2i}{z+4}\right) = \frac{(y-2)(x+4)-xy}{(x+4)^2+y^2} = \frac{4y-2x-8}{(x+4)^2+y^2}$$

$$Im\left(\frac{z-2i}{z+4}\right) = 0$$

$\therefore 4y-2x-8 = 0$ $\qquad$ $x-2y+4 = 0$



$$AN = \text{minimum } |z-1| = \left|\frac{1-2\times 0+4}{\sqrt{1^2+2^2}}\right| = \frac{5}{\sqrt{5}} = \sqrt{5}$$

3. a)

i. let $z = x + iy \quad x, y \in \mathbb{R}$

$Re\ (z) = x; \quad |Re\ (z)| = |x|$

$|z| = \sqrt{x^2 + y^2}$

$|z|^2 = x^2 + y^2, \quad |Re\ (z)|^2 = |x|^2$

$\therefore |z|^2 \geq |Re\ (z)|^2$

$|z| \geq |Re\ (z)|$ Equality when $y = 0$

ii. Let $z_1 = x_1 + iy_1$, $z_2 = x_2 + iy_2$ where $x_1, y_1, x_2, y_2 \in \mathbb{R}$

$z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$

$|z_1 + z_2|^2 = (x_1 + x_2)^2 + (y_1 + y_2)^2$

$= (x_1{}^2 + y_1{}^2) + (x_2{}^2 + y_2{}^2) + 2(x_1x_2 - y_1y_2)$

$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2Re\ (z_1\overline{z_2})$

or

$|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2Re\ (z_2\overline{z_1})$

iii. $\{|z_1| + |z_2|\}^2 = |z_1|^2 + |z_2|^2 + 2|z_1||z_2|$

$\{|z_1| + |z_2|\}^2 = |z_1 + z_2|^2 - 2Re\ (z_1\overline{z_2}) + 2|z_1||z_2|$

$\{|z_1| + |z_2|\}^2 - |z_1 + z_2|^2 = 2\{Re\ (z_1\overline{z_2}) + |z_1||z_2|\}$

$= 2\{Re\ (z_1\overline{z_2}) + |z_1||z_2|\}$

$= \left\{\sqrt{(x_1{}^2 + y_1{}^2) + (x_2{}^2 + y_2{}^2)} - (x_1x_2 - y_1y_2)\right\}$

Let $(x_1{}^2 + y_1{}^2) + (x_2{}^2 + y_2{}^2) - (x_1x_2 - y_1y_2)^2$

$\equiv (x_1y_2 - x_2y_1)^2 \geq 0$

$\sqrt{(x_1{}^2 + y_1{}^2) + (x_2{}^2 + y_2{}^2)} - (x_1x_2 - y_1y_2) \geq 0$

$|z_1||z_2| - Re\ (z_1\overline{z_2}) \geq 0$

$\{|z_1| + |z_2|\}^2 - |z_1 + z_2|^2 \geq 0$

$\{|z_1| + |z_2|\}^2 \geq |z_1 + z_2|^2$

$|z_1| + |z_2| \geq |z_1 + z_2|$

iv. $|z_1 - z_2|^2 - \{|z_1| - |z_2|\}^2 = 2|z_1||z_2| - 2Re\,(z_1\overline{z_2})$ from $(2).(a)$

$$= \left\{\sqrt{(x_1{}^2 + y_1{}^2) + (x_2{}^2 + y_2{}^2)} - (x_1x_2 - y_1y_2)\right\}$$

$$\geq 0$$

$|z_1 - z_2|^2 = \{|z_1| - |z_2|\}^2 \geq 0$

$|z_1 - z_2|^2 \geq \{|z_1| - |z_2|\}^2$

$|z_1 - z_2| \geq |z_1| - |z_2|$

$-|z_1 - z_2| \geq |z_1| - |z_2|$

b) if $|z| = 1$, then $\frac{z+1}{z^2+z-6}$

Let $z = x + iy$ where x, y are real:

$$k = \left|\frac{z+1}{z^2+z-6}\right|^2 = \frac{|z+1|^2}{|z^2+z-6|^2} = \frac{|z+1|^2}{|z+3|^2|z-2|^2}$$

$|z+1|^2 = |x + iy \ + 1|^2 = (x+1)^2 + y^2 = x^2 + y^2 + 1 + 2x$

$= 2(1+x)$ since $|z| = 1 \Leftrightarrow x^2 + y^2 = 1$

$|z+3|^2 = |x + iy \ + 3|^2 = (x+3)^2 + y^2 = x^2 + y^2 + 9 + 6x$

$= 10 + 6x = 2(5 + 3x)$

$|z-2|^2 = |x + iy \ - 2|^2 = (x-2)^2 + y^2 = x^2 + y^2 + 4 - 4x$

$= (5 - 4x)$

$$k = \frac{2(1+x)}{2(5+3x)(5-4x)} = \frac{(1+x)}{(5+3x)(5-4x)}$$

$k(5+3x)(5-4x) = 1 + x$

$-12kx^2 - 5kx + 25k = 1 + x$

$-12kx^2 - (1+5k)x - (25k - 1) = 0$

Since $x \in \mathbb{R}$ $\quad (1+5k)^2 + 48(25k-1) \quad \geq 0$

$1225k^2 - 38k + 19 \geq 0$

$$k^2 - \frac{38}{1225}k + \left(\frac{19}{1225}\right)^2 \geq \frac{19^2}{1225^2} - \frac{1}{1225}$$

$$\left(k - \frac{19}{1225}\right)^2 \geq \frac{-864}{1225^2}$$

$\therefore$ k can have any value

$\therefore$ $\left|\frac{z+1}{z^2+z-6}\right|$ have all real values.

4. a)

i)

$z = \cos\theta + \sin\theta$

$z^n = (\cos\theta + \sin\theta)^n = \cos n\theta + \sin n\theta$

$\therefore (\cos\theta + \mathrm{i}\sin\theta)^4 = \cos 4\theta + \sin 4\theta --- (1)$

$(\cos\theta + \sin\theta)^4 = \cos^4\theta + 4i\cos^3\theta\sin\theta - 6\cos^2\theta\sin^2\theta$

$-4i\cos\theta\sin^3\theta + \sin^4 4 --- (2)$

Equating real part of (1) and (2)

$\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$

$\frac{\cos 4\theta}{\cos^4\theta} = 1 - 6\tan^2\theta + \tan^4\theta \qquad \left(\cos^4\theta \neq 0, \quad \theta = (2n+1)\frac{\pi}{4}\right)$

If

$\theta = \frac{\pi}{8} \qquad \cos\frac{\pi}{8} \times 4 = \cos\frac{\pi}{2} = 0$

$\therefore 1 - 6\tan^2\theta + \tan^4\theta = 0$

$$\tan^2\theta = \frac{6 \pm \sqrt{36-4}}{2}$$

$$= \frac{6 \pm 4\sqrt{2}}{2} = 3 \pm 2\sqrt{2}$$

$\tan^2\theta = 3 + 2\sqrt{2} = \left(\sqrt{2}+1\right)^2$

$\therefore \tan\theta = \sqrt{2} + 1 > 1$

$\theta = \frac{3\pi}{8}$

$\tan^2\theta = 3 - 2\sqrt{2} = \left(\sqrt{2}-1\right)^2$

$\tan\theta = \sqrt{2} - 1 < 1$

$$\therefore \tan\frac{\pi}{8}\theta = \sqrt{2} - 1$$

ii.

$$z = \frac{1}{2}\left(\sqrt{3} + i\right) = \frac{\sqrt{3}}{2} + \frac{i}{2} = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$$

$$\left(\frac{\sqrt{3} + i}{2}\right)^{2011} = \left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^{2011}$$

$$= \left(\cos\frac{2011\pi}{6} + i\sin\frac{2011\pi}{6}\right)$$

$$= \cos\left(167.2\,\pi + \frac{7\pi}{6}\right) + i\sin\left(167.2\,\pi + \frac{7\pi}{6}\right)$$

$$= \cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}$$

$$= -\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}$$

$$= -\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)$$

b. $z^5 = -1;$

let $z = \cos\theta + i\sin\theta$

$z^5 = (\cos 5\theta + i\sin 5\theta)$

$-1 = \cos(2k\pi + \pi) + \sin(2k\pi + \pi)$

$\cos 5\theta + i\sin 5\theta = \cos(2k + 1)\pi + \sin(2k + 1)\,\pi$

$\therefore 5\theta = (2k + 1)\pi$

$\theta = \frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5}$

The fifth root of (-1) are

$$z_1 = \cos\frac{\pi}{5} + i\sin\frac{\pi}{5}$$

$$z_2 = \cos\frac{3\pi}{5} + i\sin\frac{3\pi}{5}$$

$$z_3 = \cos\pi + i\sin\pi$$

$$z_4 = \cos\frac{7\pi}{5} + i\sin\frac{7\pi}{5}$$

$$z_5 = \cos\frac{9\pi}{5} + i\sin\frac{9\pi}{5}$$

$$z^5 + 1 = z + 1[z^4 - z^3 + z^2 - z + 1]$$

$$z + 1 = 0 \qquad z = -1, \;\; z = \cos\pi + i\sin\pi$$

$\therefore$ The root of $z^4 - z^3 + z^2 - z + 1 = 0$ are z_1, z_2, z_3, z_4

We know that

$$z_3 = z_1 = \cos\frac{\pi}{5} - i\sin\frac{\pi}{5}$$

$$z_4 = z_2 = \cos\frac{3\pi}{5} - i\sin\frac{3\pi}{5}$$

$$\therefore z_1 + z_2 + z_3 + z_4 = 2\left(\cos\frac{\pi}{5} + i\sin\frac{\pi}{5}\right) = 1$$

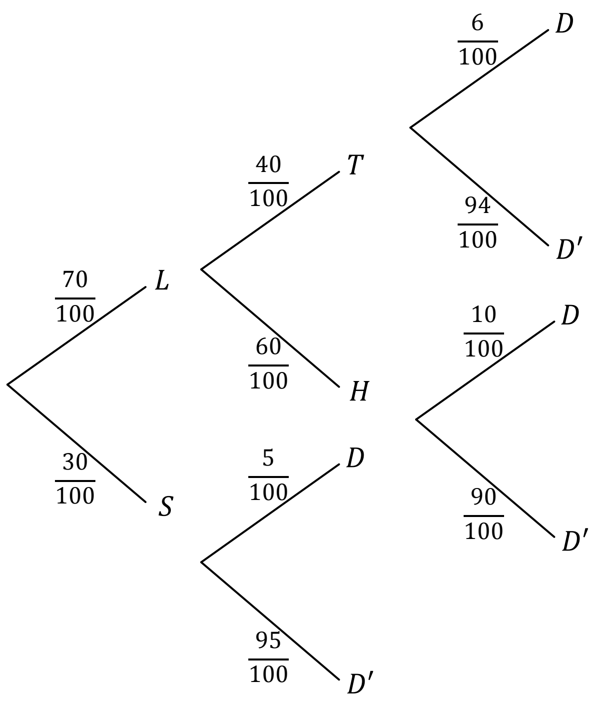
Since $\alpha, \beta, \gamma, \delta$ are the root of the equation

$$ax^4 + bx^3 + cx^2 + dx + e = 0$$

$$\therefore \alpha + \beta + \gamma + \delta = \frac{-b}{a}$$

$$\therefore \cos\frac{\pi}{5} + i\sin\frac{\pi}{5} = \frac{1}{2}$$

5. a.



$$p(D)=\frac{70}{100}\times\frac{40}{100}\times\frac{6}{100}+\frac{70}{100}\times\frac{60}{100}\times\frac{10}{100}+\frac{30}{100}\times\frac{5}{100}$$

$$=\frac{168+420+150}{1000}=\frac{738}{10000}=\frac{369}{5000}$$

A.

$$P\left({}^{L}/_{D}\right)=\frac{P\left({}^{D}/_{L}\right).P(L)}{P(D)}$$

$$=\frac{\frac{70}{100}\times\left(\frac{40}{100}\times\frac{6}{100}+\frac{60}{100}\times\frac{10}{100}\right)}{\frac{369}{5000}}$$

$$=\frac{70}{100}\times\frac{840}{10000}\times\frac{5000}{369}$$

$$=\frac{588}{738}=0.796$$

B.

$$P\left({}^{S}/_{D}\right)=\frac{\frac{30}{100}\times\frac{5}{100}}{\frac{369}{5000}}$$

$$=\frac{30}{100}\times\frac{5}{100}\times\frac{5000}{369}$$

$$=\frac{75}{369}=0.203$$

C.

$$P\left({}^{T}/_{D}\right)=\frac{\frac{70}{100}\times\frac{40}{100}\times\frac{6}{100}}{\frac{369}{5000}}$$

$$=\frac{70\times 40\times 6}{1000000}\times\frac{5000}{369}$$

$$=\frac{84}{369}=0.227$$

b. i)

$$f(y)=\begin{cases}cy & 0\le y\le 2\\ 0 & o/w\end{cases}$$

Since $f(y)$ is pdf $\int_{-\infty}^{\infty}f(y)dy=1$

ii)

$$\int_0^2 cy\,dy = \left[c\frac{y^2}{2}\right]_0^2 = \frac{c \times 4}{2} = 1$$

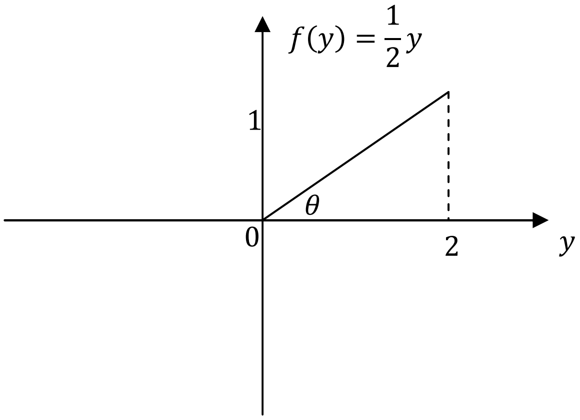
$$2c = 1$$

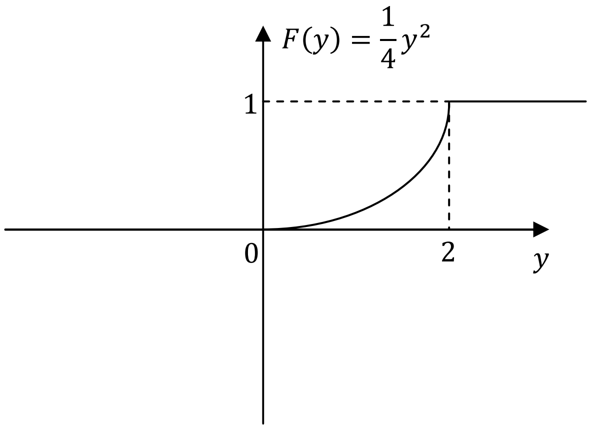
$$c = \frac{1}{2}$$

$$F(y) = \int_{-\infty}^{y} cy\,dy = \int_{-\infty}^{y} \frac{1}{2}y\,dy = \left[\frac{y^2}{4}\right]_0^y$$

$$F(y) = \tfrac{1}{4}y^2$$

iii)





iv)

$$E(y) = \int_{-\infty}^{\infty} yf(y)\,dy = \int_0^2 y\frac{y}{2}\,dy = \int_0^2 \frac{y^2}{2}dy = \left[\frac{y^3}{3}\right]_0^2$$

$$= \frac{8}{6} = \frac{4}{3}$$

$$E(y^2) = \int_0^2 \frac{y^3}{2}dy = \left[\frac{y^4}{8}\right]_0^2$$

$$= \frac{16}{8} = 2$$

$$V(y) = E(y^2) - [E(y)]^2$$

$$= 2 - \left(\frac{4}{3}\right)^2 = \frac{2}{9}$$

6. a) probability of the fish that can be saved $(P) = 20\%$,

Total number of fish $(n) = 20$

i) $P(n = 14) = \ {}^{20}C_{14}(0.8)^{14}(0.2)^6$

$$= 0.10909$$

ii) $P(n \geq 10) = \ {}^{20}C_{10}(0.8)^{10}(0.2)^{10} + \ {}^{20}C_{11}(0.8)^{11}(0.2)^9 + \ {}^{20}C_{12}(0.8)^{12}(0.2)^8$

$+ {}^{20}C_{13}(0.8)^{13}(0.2)^7 + \ {}^{20}C_{14}(0.8)^{14}(0.2)^6 + \ {}^{20}C_{15}(0.8)^{15}(0.2)^5$

$+ \ {}^{20}C_{16}(0.8)^{16}(0.2)^4 + \ {}^{20}C_{17}(0.8)^{17}(0.2)^3 + \ {}^{20}C_{18}(0.8)^{18}(0.2)^2$

$+ \ {}^{20}C_{19}(0.8)^{19}(0.2)^1 + {}^{20}C_{20}(0.8)^{20}(0.2)^0$

$$= 0.99944$$

iii) $P(n \leq 16) = 1 - P(n > 16) = {}^{20}\ C_{17}(0.8)^{17}(0.2)^3 + \ {}^{20}C_{18}(0.8)^{18}(0.2)^2 +$

${}^{20}C_{19}(0.8)^{19}(0.2)^1 + {}^{20}\ C_{20}(0.8)^{20}(0.2)^0$

$$= 0.58855$$

b)

i)

$$P(r) = \frac{e^{-m}m^r}{r!}$$

$$\mathrm{P}(x \leq 3) = p(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)$$

$$P(x \leq 3) = \frac{e^{-7}7^0}{0!} + \frac{e^{-7}7^1}{1!} + \frac{e^{-7}7^2}{2!} + \frac{e^{-7}7^3}{3!}$$
$$= e^{-7}\left[1 + \frac{7}{1} + \frac{49}{2} + \frac{343}{6}\right]$$
$$= 0.0817$$

ii)

$$P(x \geq 2) = p(x = 2) + P(x = 3) + P(x = 4) + P(x = 5) + P(x = 6) + P(x = 7)$$

$$\mathrm{P}(x \leq 2) = 1 - p(x < 2) = 1 - P(x = 0) + P(x = 1)$$

$$P(x \geq 2) = \frac{e^{-7}7^0}{0!} + \frac{e^{-7}7^1}{1!}$$

$$= 1 - e^{-7}\left[1 + \frac{7}{1}\right]$$

$$= 0.9927$$

(iii)

$$P(x = 5) = \frac{e^{-7}7^5}{5!}$$

$$= 0.1277$$